

# A study on elasto-plastic behavior of one-twentieth scale reinforced concrete frames

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## ABSTRACT

This paper describes the structural behavior of the reinforced concrete frames based on the experiments and analyses on the one-twentieth scale models. Focusing on the influence of the beam elongation, the experimental program consists of monotonic loading tests on the four-story reinforced concrete frames with one, three or seven spans. The structural behavior of the frames was also analyzed by the finite segment method.

As conclusions obtained from the experiments and analyses, (1) the horizontal displacement vs. horizontal load relationship for each column in the frames was not identical due to the axial elongation of bending yield beams, (2) the lateral loads for all the frames was increased due to the axial restriction in beams caused by the lateral stiffness of columns, (3) the results of the analysis showed a good agreement with the experimental results.

## INTRODUCTION

Reinforced concrete beams and columns forming up a structural frame exert influence on each other during loading and behave physically as one structural body.

Thus, whenever analysis of the mechanical behavior of the reinforced concrete frame is concerned, experiment carried out on the frame as a whole system makes more sense than just experimenting on the individual member.

A loading test of a statically indeterminate reinforced concrete beam-to-column connection subassemblies was carried out by Zerbe and Durrani(1988), who pointed out that compressive force was developed in a beam which subsequently increased its flexural strength. Focusing on the restraint of the beam elongation, the author and his colleagues studied the behavior of the reinforced concrete beam subjected to the restraint of axial deformation(Kokusho, Hayashi, Wada and Sakata 1988).

In order to clarify the influence of the axial elongation of beams, horizontal loading experiments and analyses of a beam sidesway mechanism type (with the number of spans used as a parameter), multi-story multi-span reinforced concrete plane frame were carried out. In this paper, the axial elongation of beams, the axial strain developed in the beams, the horizontal strength of the frame, the behavior of collapse mechanism, and etc. will be discussed.

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## METHODS OF EXPERIMENTS

### Specimens

With reference to a certain reinforced concrete building with a pure rigid frame, a 1/20 scale plane frame was envisaged representing the lower portion of the building, and a total of three specimens; a four-story, one-span specimen (FR1), a four-story, three-span specimen (FR3) and a four-story, seven-span specimen (FR7) were manufactured. Each specimen represents the portion up to the intermediate height of the columns on the fifth floor erected on the fourth story of the specimen. The cross section of the members is shown in Fig.1. The dimension of the FR7 specimen is shown in Fig.2. For the purpose of later explanation, the columns are marked to show A, B, C, D, E, F, G and H in order from the left to the right.

In lieu of concrete, the mortar with cement and standard sand proportioned at mixing ratio 1:2.5, which was mixed with water at a water-cement ratio of 75%, was used. D3 deformed reinforcement (Murayama et al. 1982) used as main reinforcement was assembled with spiral hoops and stirrups manufactured from a 1.6mm diameter annealed wire.

### Loading apparatus

The loading apparatus and the method of loading are shown in Photo1. and Fig.3.

<Axial force of column> : Fig.3 shows the method of introducing the axial force of column, (N). The plate attached to the top of the column of each specimen was connected to the spring located in the spring case underneath the reaction frame, by means of a round steel bar with pin-shaped ends. By tightening the nut underneath the spring, the vertical force of 13.7kN per column was caused to act as a fixed load.

<Horizontal force> : Fig.3 shows the method of applying a monotonic, horizontal force. A pin was fitted with each top of column, and a mini oil jack was installed between the pin and the

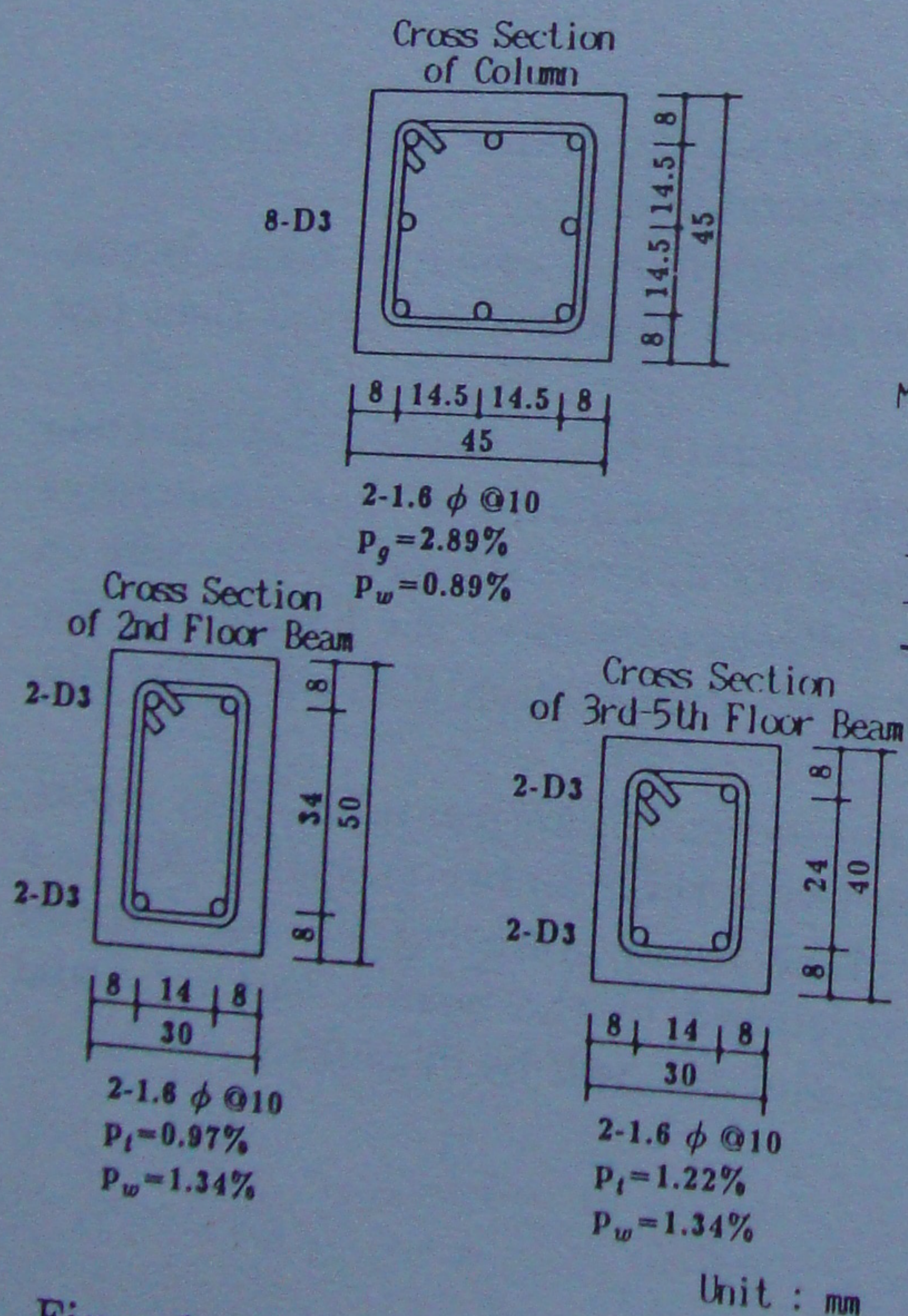


Figure1. Cross Section of Specimens

Table1. Physical Properties of Materials

Reinforcing Bar				
	Sectional Area (cm <sup>2</sup> )	Yield Strength (MPa)	Tensile Strength (MPa)	Elongation (%)
D3	0.0731	317.3	440.9	40.7
1.6φ	0.0201	253.7	383.8	

Mortar				
Material Age (Day)			Exp. Start	Exp. End
7	28	40	56	
Compressive Strength (MPa)		21.5	32.2	40.1
Strain at Compressive Strength (×10 <sup>-6</sup> )			3526	3896
Young's Modulus (MPa)			21379	22948
Tensile Strength (MPa)	1.8	2.4	3.0	3.4

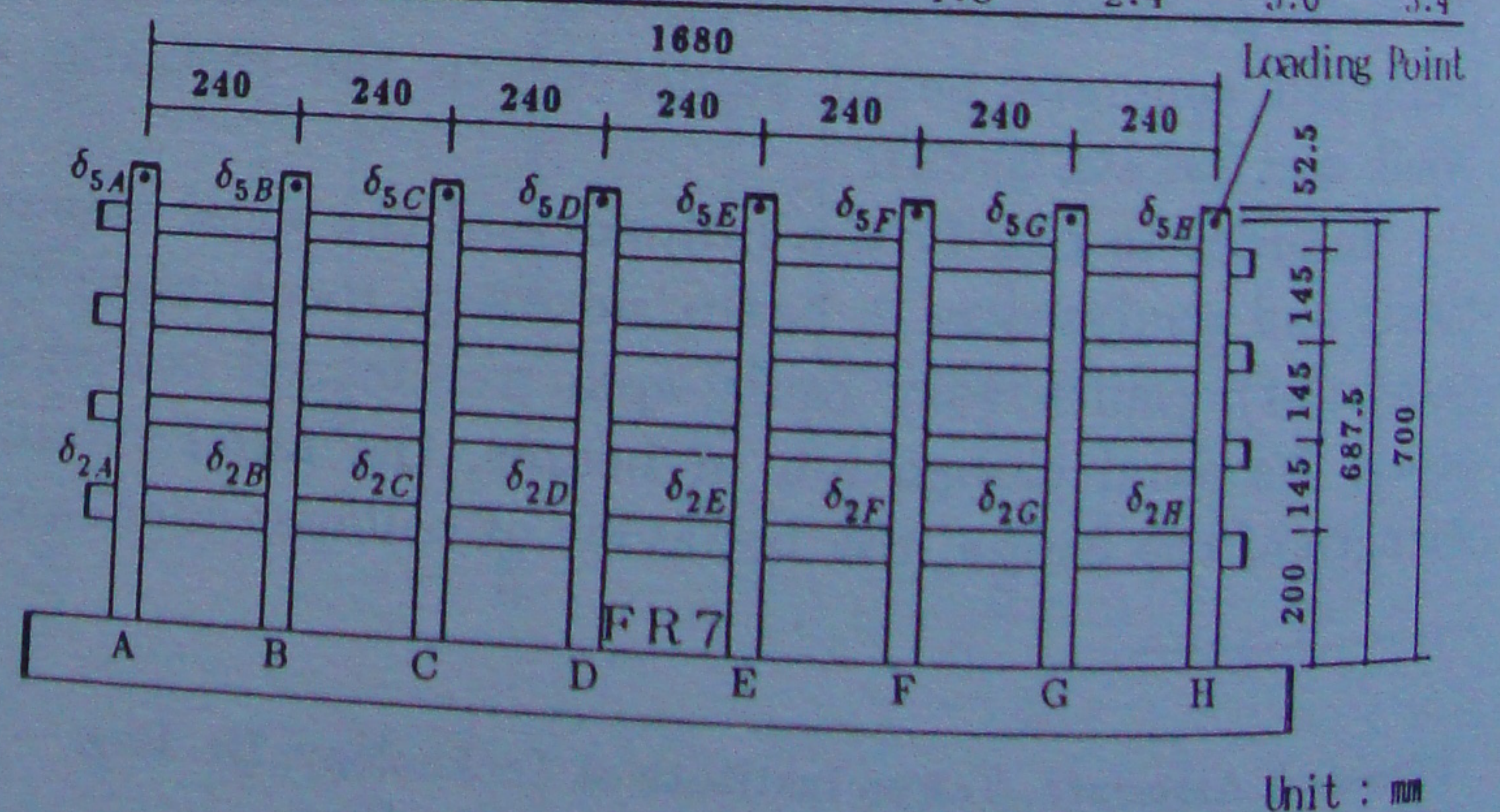


Figure2. Specimens (FR7)



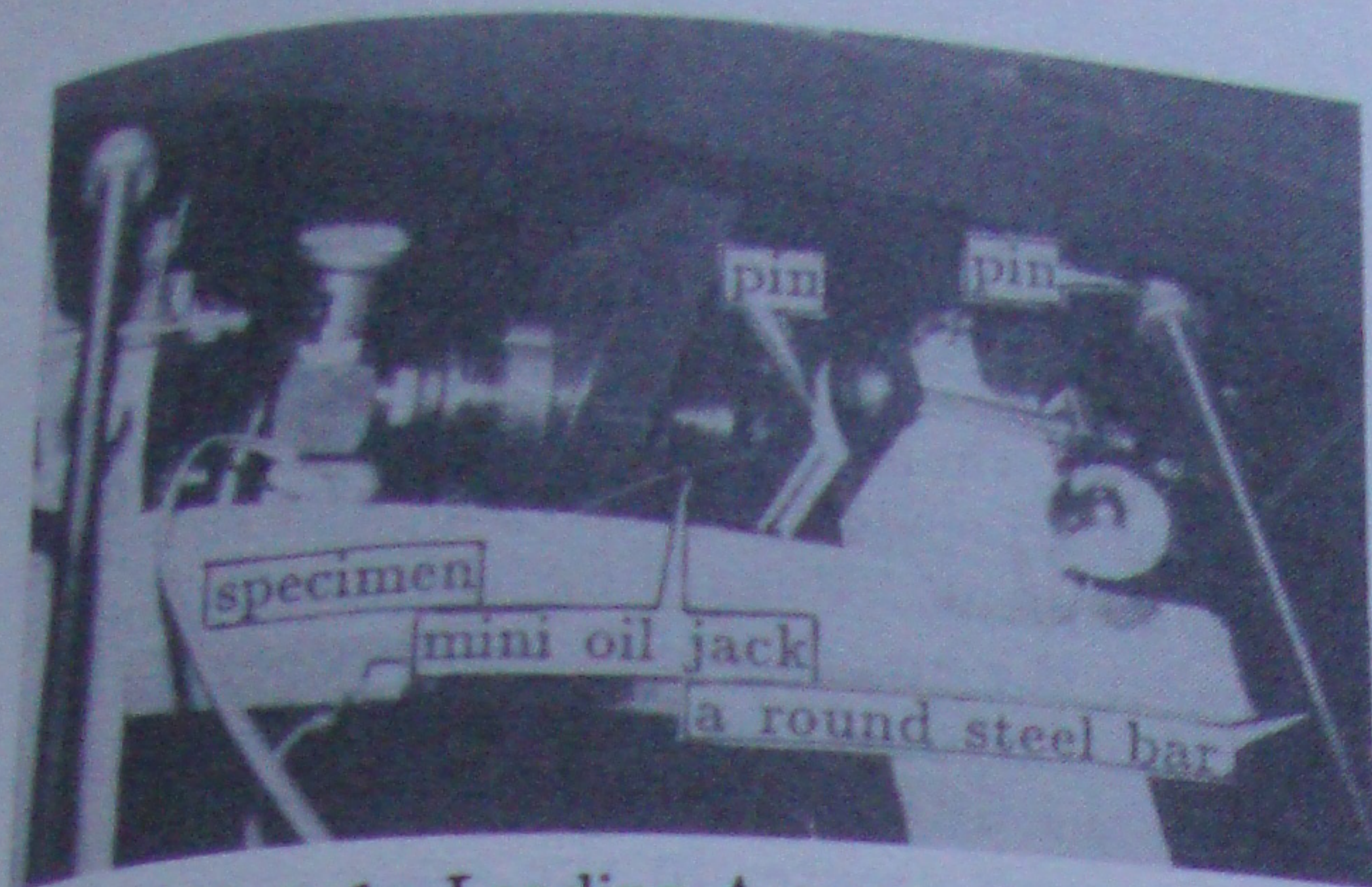


Photo1. Loading Arrangement

Table2. Horizontal Resistance Force Calculated by Simple Analysis Assuming Mechanism of Collapse

FR1	FR3	FR7
3.53kN	9.12kN	20.40kN

loading frame. These mini oil jacks were connected with one unit of hydraulic pump, and are capable of applying force uniformly to each top of column. However, in order for the columns at both ends to be subject to half the force in other columns, a mini oil jack was used with a cross section being half that of another mini oil jack.

#### Mechanical properties of materials

Table1 shows the mechanical properties of the D3 deformed reinforcement, the 1.6mm diameter annealed wire, and the mortar which were used.

#### Horizontal resistance force calculated by simple analysis assuming mechanism of collapse

Table2 shows the estimated horizontal resistance force of the specimens, which were obtained on the assumption that no axial force is developed in the beams of a collapse mechanism with a yield hinge assumed at the beam ends and column bases. For this model, consideration was given to the rigid zone in the beam ends.

## RESULTS OF EXPERIMENTS AND DISCUSSIONS

#### Relationship between horizontal force and horizontal displacement

Fig.4 shows the relationship between horizontal force and horizontal displacement. In the figure, symbols such as  $\delta_{5A}$ ,  $\delta_{5B}$ , ...,  $\delta_{5H}$  are used corresponding with those shown in Fig.2. The value in parentheses denotes the angle of rotation. In the figure, the simple analysis represented by broken line considers the N- $\delta$  effect of the horizontal resistance force in Table2.

In all specimens, the horizontal displacements of all the columns on each floor were identical in an initial stage, which began to vary after initial cracking occurred, and the horizontal displacements of the columns on the left became more noticeable than the columns on the right when the whole average angle of rotation exceeded 1/200. This was due to the cumulative axial elongation of the flexuously yielded beams. Each specimen indicates a greater strength than simple analysis. The reason for such increase in strength would be that the rigidity of the columns restrained the axial elongation of the flexuously yielded beams which consequently led

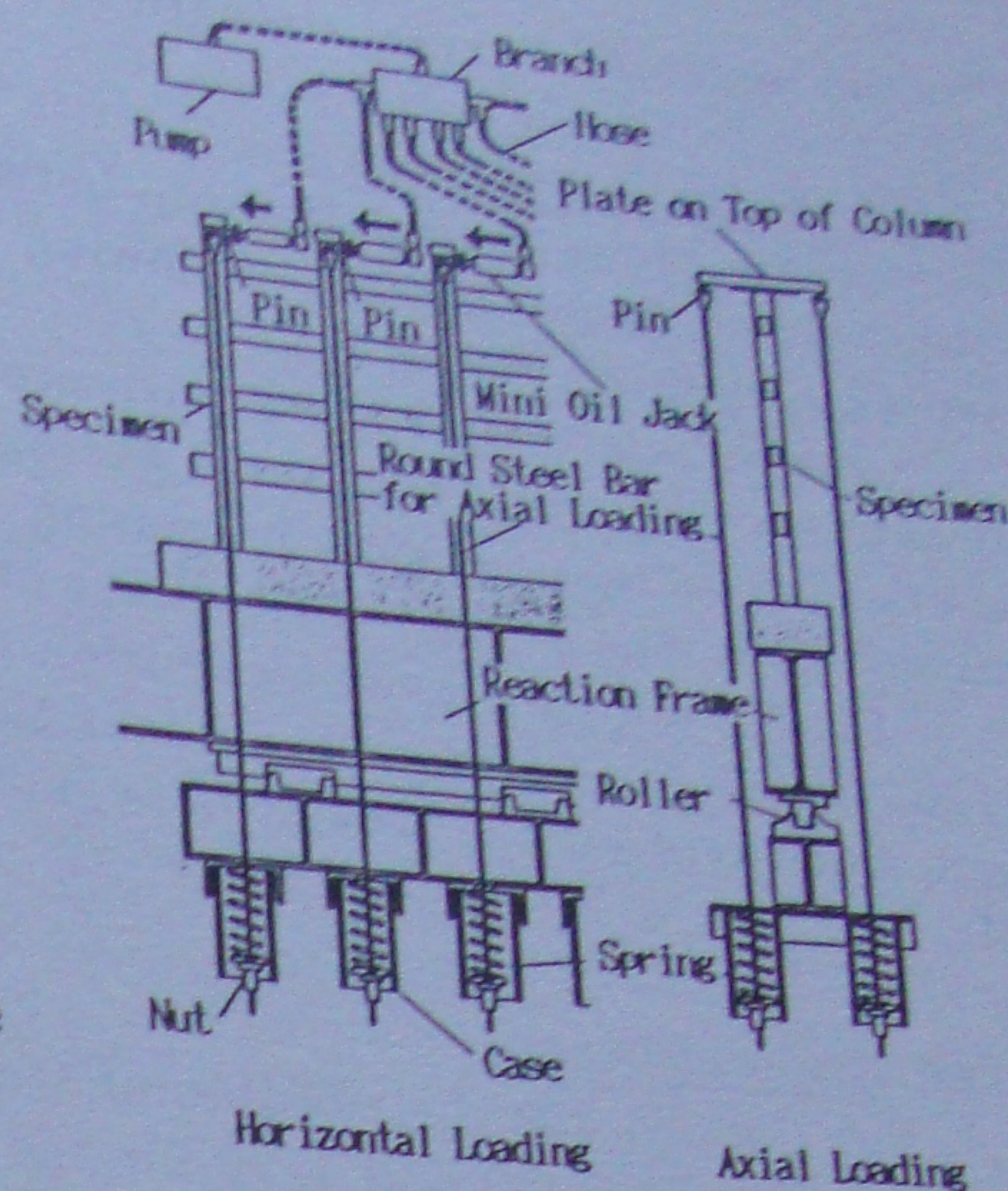


Figure3. Loading Arrangement



to the increased flexural strength of the beams.

Axial elongation of beam

Fig.5 shows the relationship between horizontal force and the axial elongation of beams on the second floor. AB ..... following  $\Delta$  represents the difference of horizontal displacement between columns A and B .....

Immediately after the occurrence of flexural cracking, the axial elongation was small, and began growing when the whole average angle of rotation exceeded some 1/200, and thereafter continued increasing with increasing horizontal displacement. Regarding the beams on the second floor, the greater the number of spans, the smaller the amount of elongation per span.

Fig.6 shows the axial elongation of the beams when the whole average angle of rotation is 1/20. From the figure, it is evident that the beams on the fifth floor indicate irregular elongation, but those on the second floor indicate elongation, much greater in the spans closer to the left.

Axial strain of beams

Fig.7 shows the distribution of strains in the beams when the whole average angle of rotation were 1/100, 1/50 and 1/20 for the FR3. Compressive strains were developed in the beams on the second floor; a maximum 50 $\mu$ , maximum 40 $\mu$  in FR1 and maximum 350 $\mu$  in FR7, indicating that the greater the number of spans, the much greater the compressive strains in the beams on the second floor.

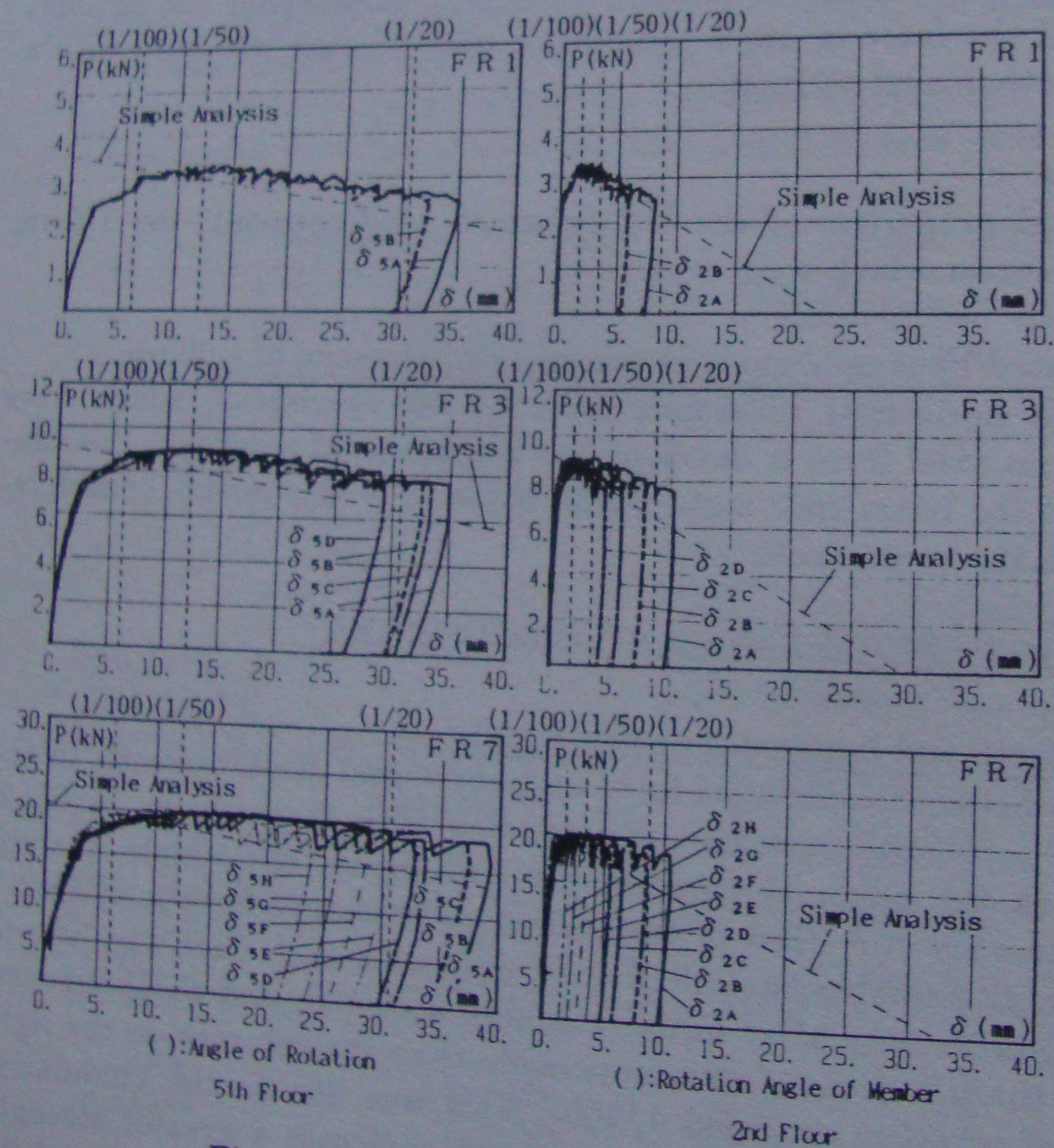


Figure 4. Horizontal Force - Horizontal Displacement Relationships

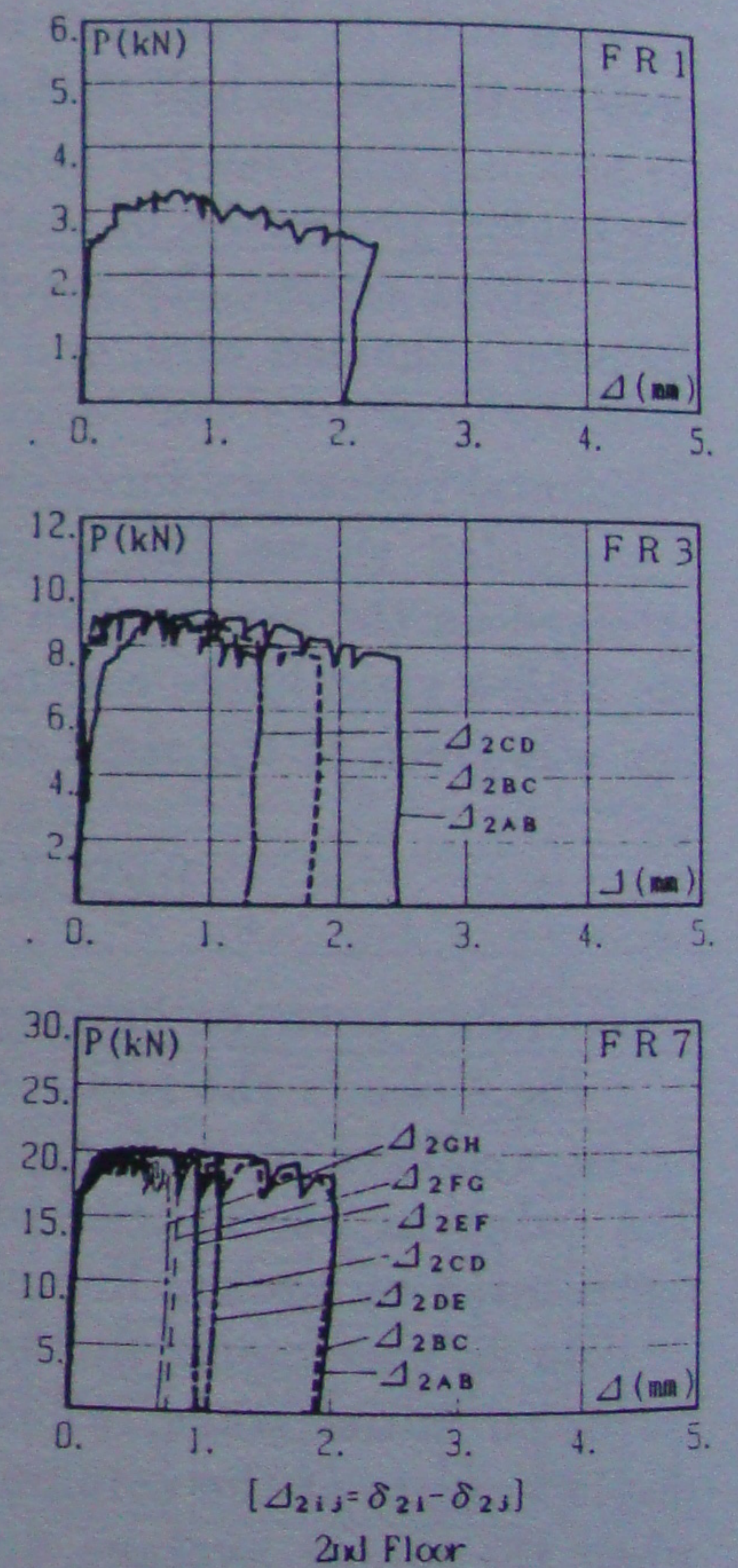


Figure 5. Horizontal Force - Axial Elongation of Beam Relationships



State of deformation

Fig.8 shows the state of ultimate deformation of the FR3. In FR1, both columns A and B indicate the mode of deformation. Columns closer to the left indicate nearly straight deformation at the positions of the beams on the second floor. From the figure, it is evident that the second floor, indicating the effect of axial elongation of the beams on the second floor also. This phenomenon is shown in FR1 and FR7.

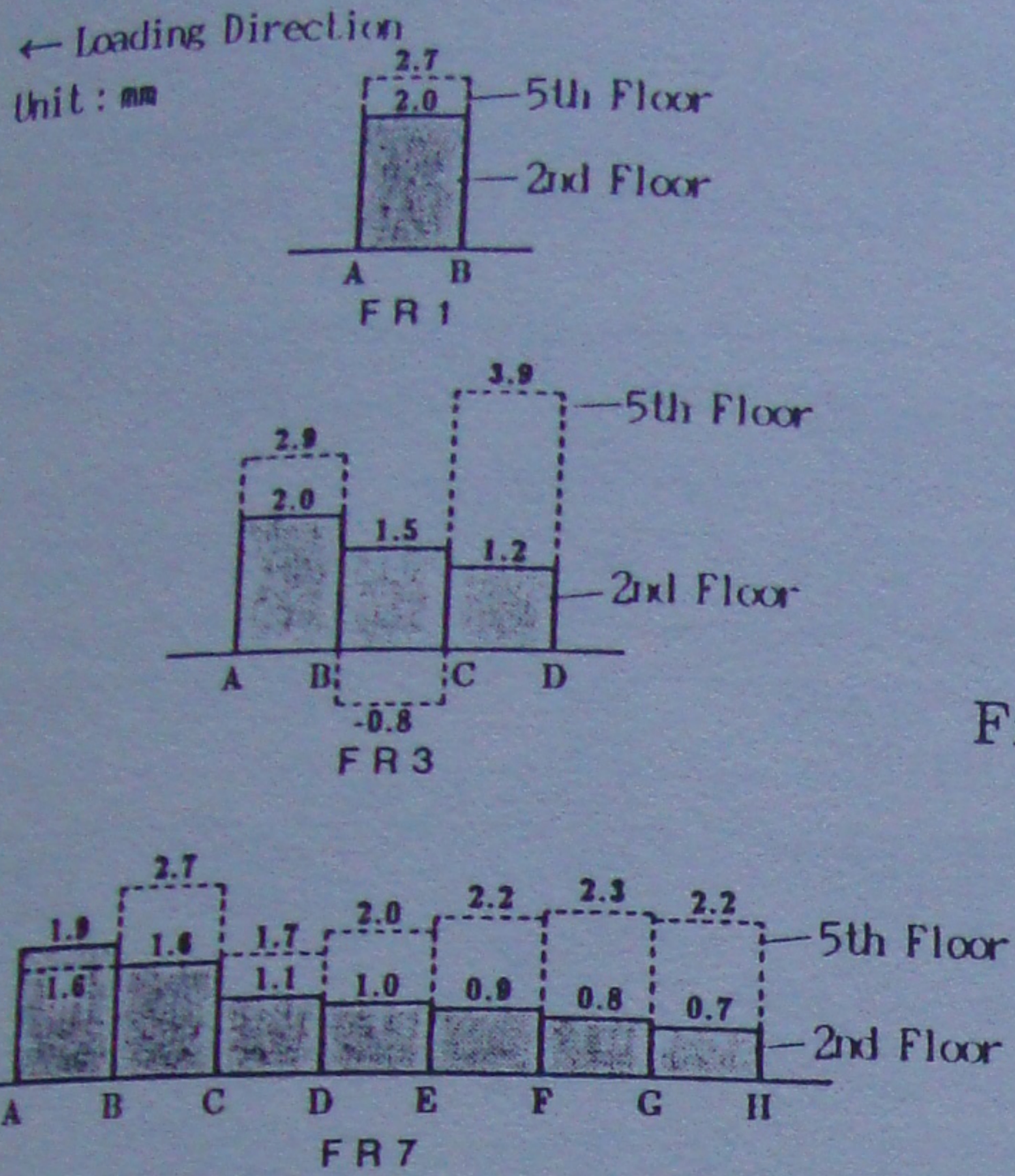


Figure6. Axial Elongation of Beam at Whole Average Angle of Rotation 1/20

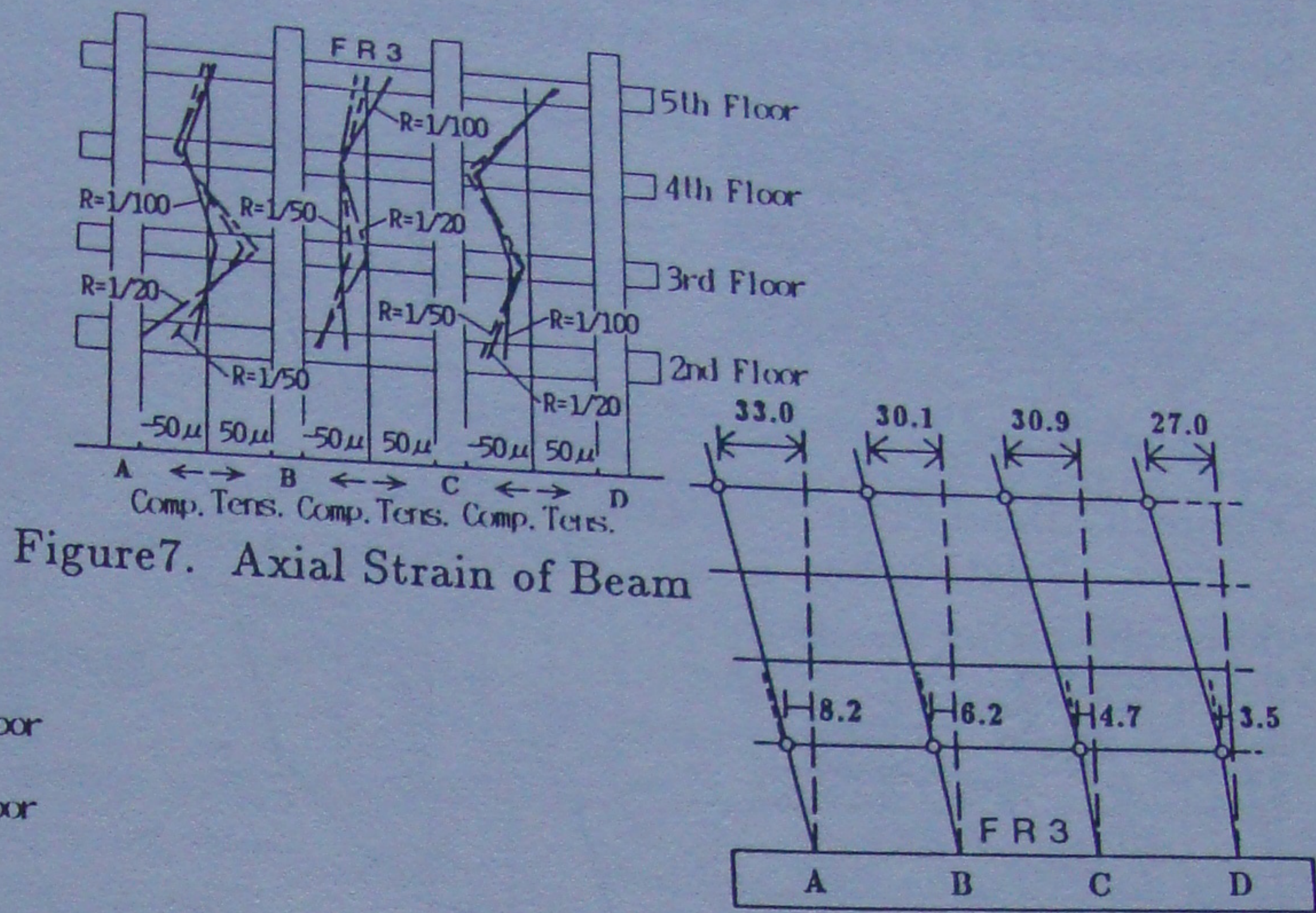


Figure8. Deformation of FR3 at Whole Average Angle of Rotation 1/20

ANALYSIS METHOD AND ANALYSIS MODEL

By the implementation of the elasto-plastic analysis method of reinforced concrete frame taking into consideration shear deformation of beam-column joints and bond-slip using finite segment method, analyses of the present experiments were carried out. As the details of the elasto-plastic analysis method are given in Reference (Kokusho, Wada and Sakata 1988), explanation of the analysis method is omitted.

Analysis model

Fig.9 shows the model for the FR3 used in analyses. Each of the columns and beams between the panels was divided into 20 elements in the direction of its axis, the columns on the fifth floor were divided into 10 elements, and the cantilever beams projecting on the left and the right were divided into 5 elements. The overall degree of freedom of the analysis models was 3722 in FR1, 7724 in FR3, and 15700 in FR7.

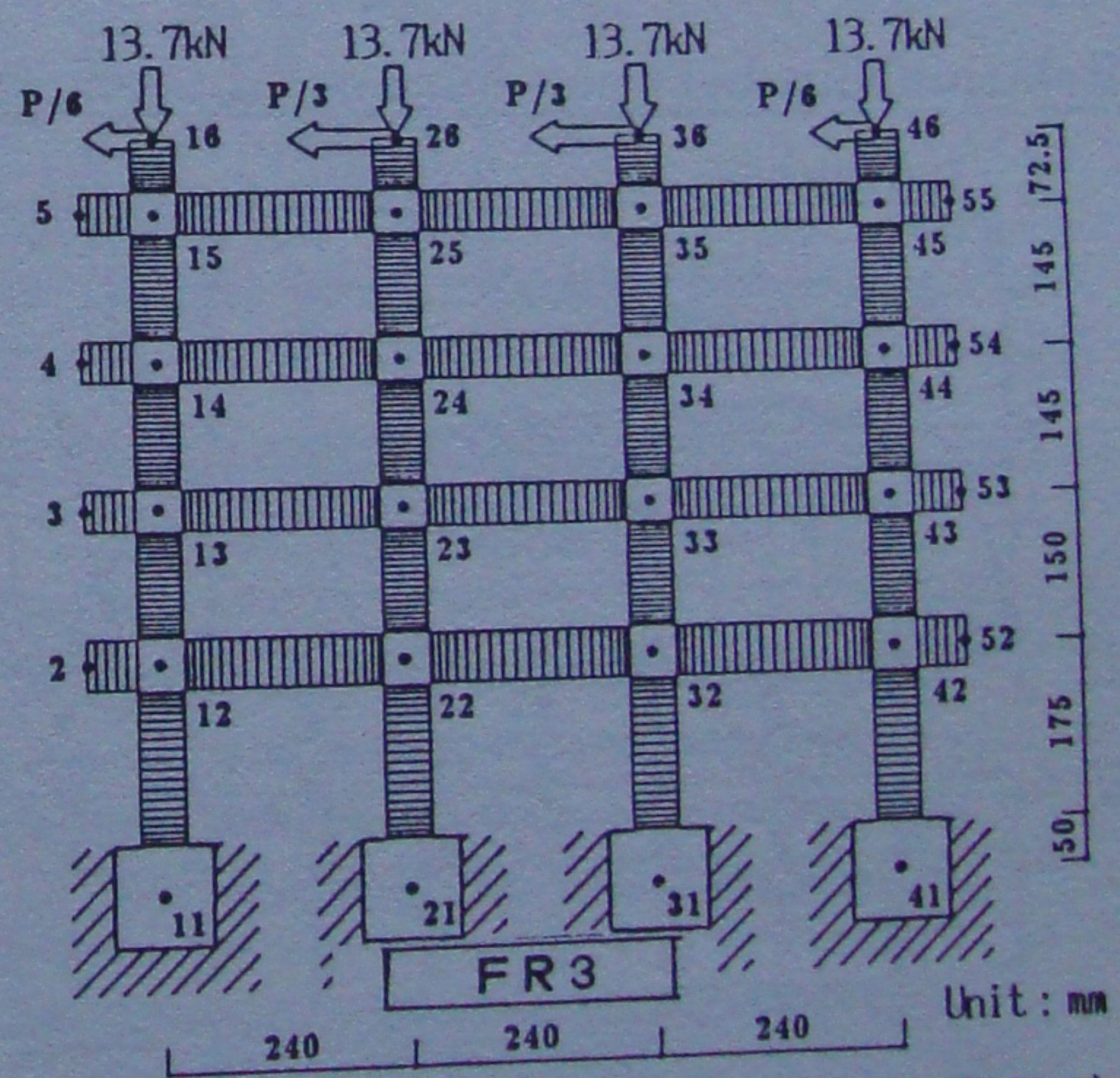


Figure9. Model Used in Analysis (FR3)



Mechanical properties of materials used in the analyses

Table 3 shows the constants used in the analyses in correspondence to the relationship between stress and strain of concrete (Fig. 11), between bond stress and slip (Fig. 10), between stress and strain of reinforcing bar (Fig. 12), and between stress and strain of concrete panel (Fig. 13). The values obtained through material tests were applied as the values of the compressive strength of concrete, the yield stress of the reinforcement, etc., and the values involved in the bond slip of the reinforcement were modeled (Fig. 10) with reference to the results of bond tests conducted by Murayama, et al. (1982).

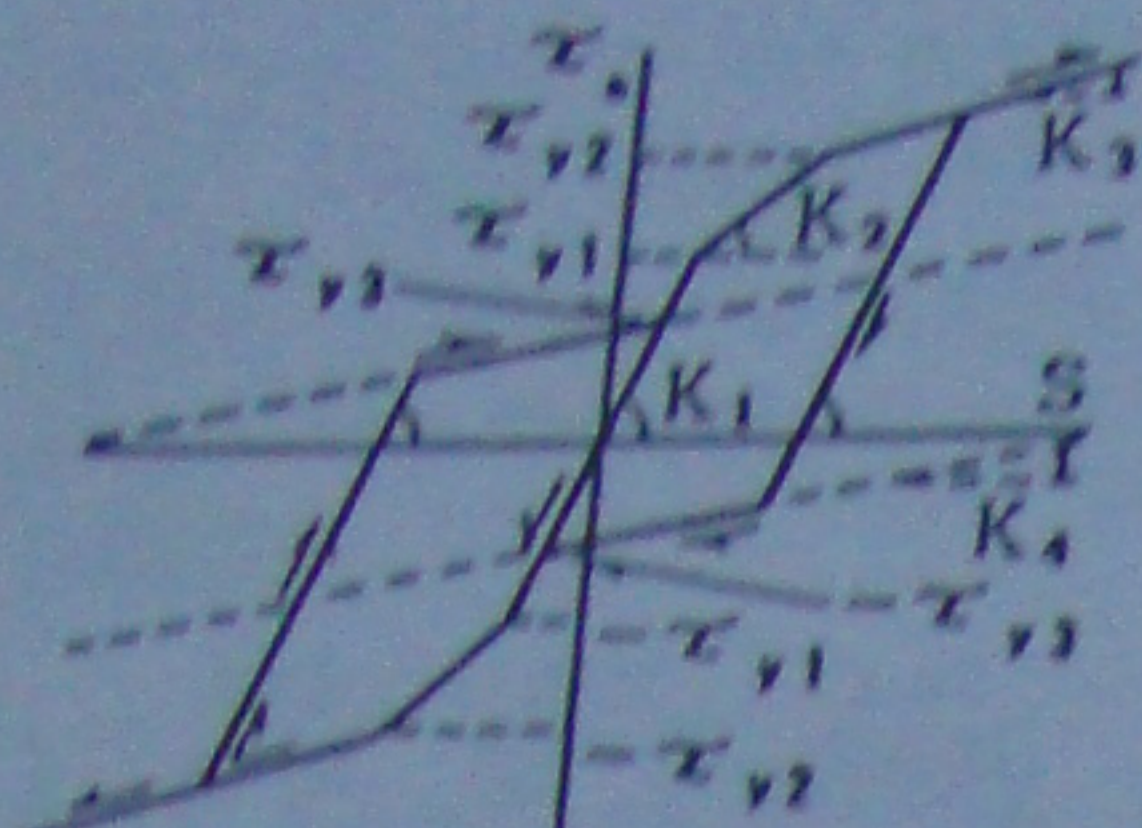


Figure 10. Bond - Slip Relationship

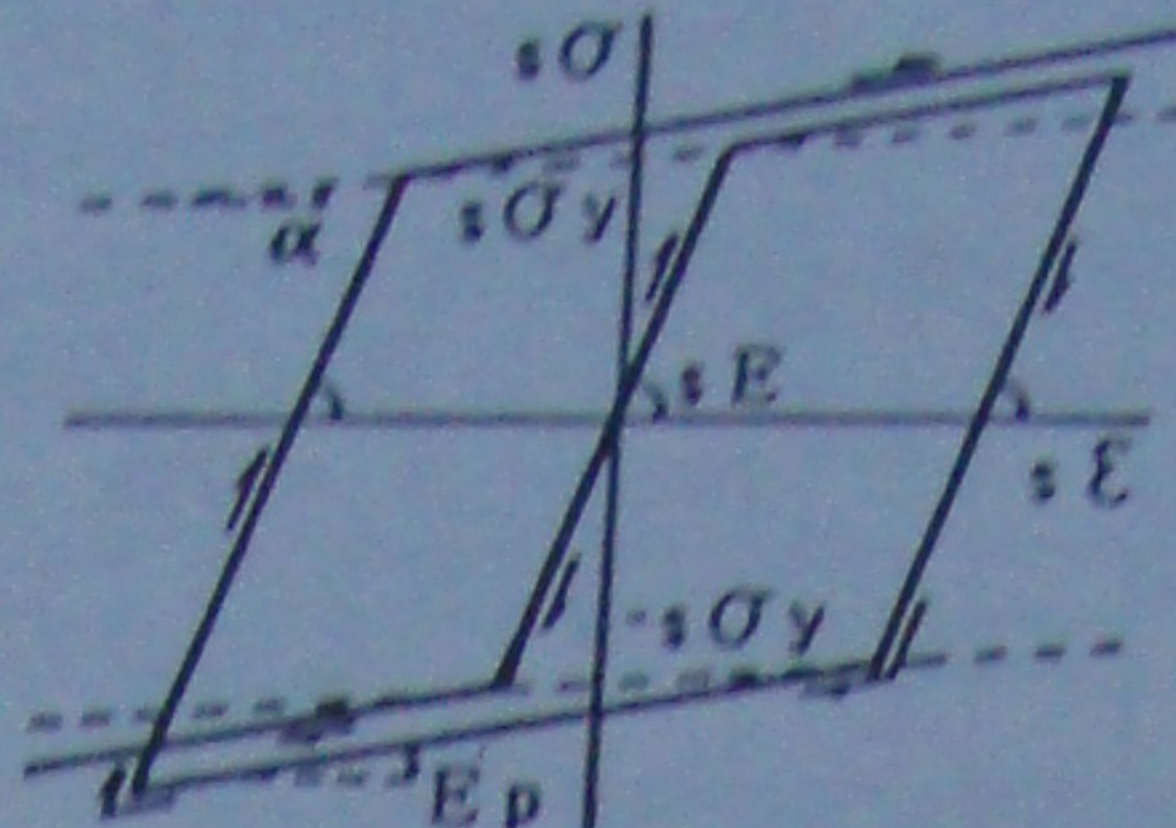


Figure 12. Stress - Strain Relationship of Rebar

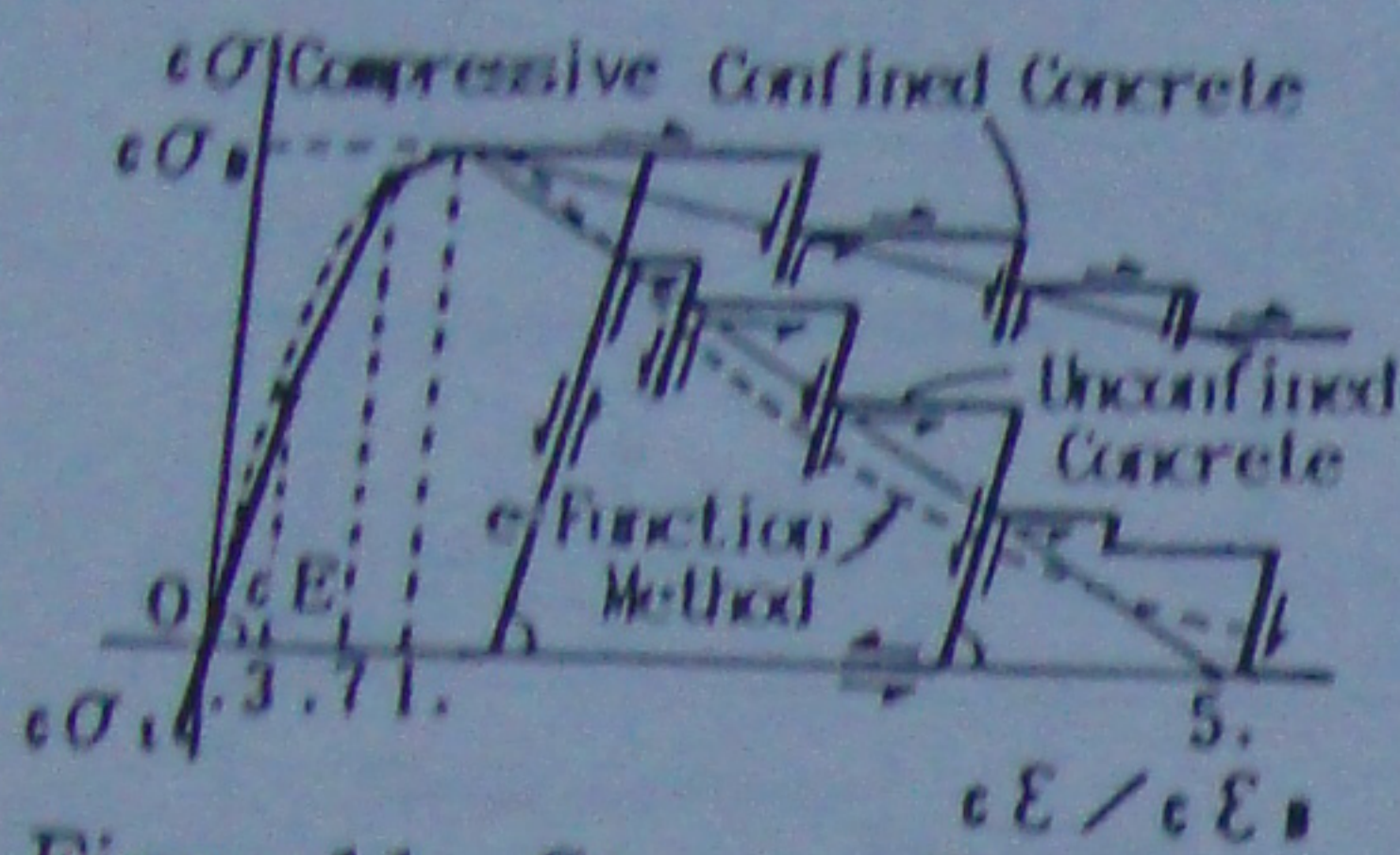


Figure 11. Stress - Strain Relationships of Concrete

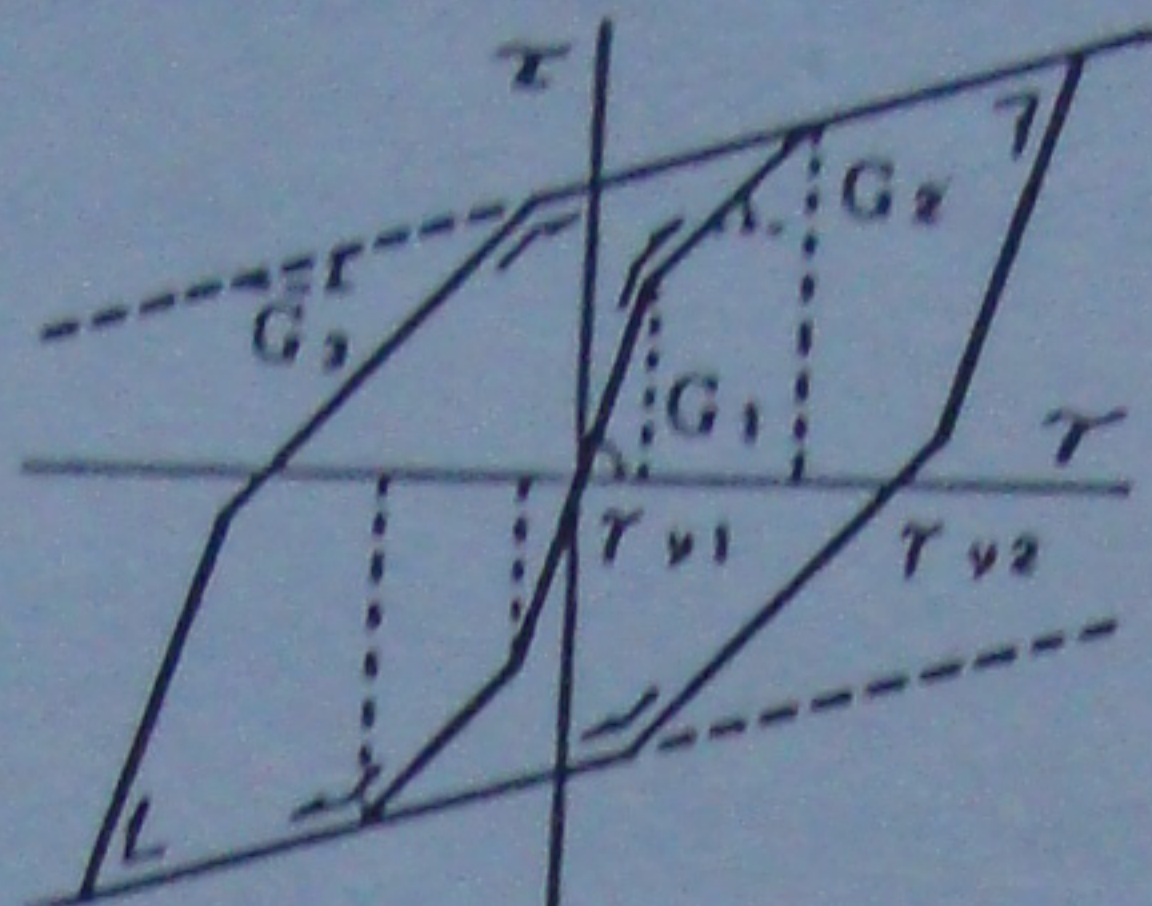


Figure 13. Stress - Strain Relationship of Concrete Panel

Table 3. Mechanical Properties of Materials Used in The Analysis

FR 1			
$\sigma_c E = 21379.$	MPa	$\sigma_c \epsilon_B = 0.0035$	
$\sigma_c \sigma_f = 40.1$	MPa		
FR 3			
$\sigma_c E = 22163.$	MPa	$\sigma_c \epsilon_B = 0.0037$	
$\sigma_c \sigma_B = 42.2$	MPa		
FR 7			
$\sigma_c E = 22948.$	MPa	$\sigma_c \epsilon_B = 0.0039$	
$\sigma_c \sigma_B = 44.2$	MPa		
Common to All			
$\sigma E = 196133.$	MPa	$\alpha = 49.$	MPa
$E_p = 98.1$	MPa	$\sigma \sigma_y = 317.3$	MPa
$K_1 = 343. (1118.)$	MPa/cm	$G_1 = 5884.$	MPa
$K_2 = 58.8 (215.7)$	MPa/cm	$G_2 = 4707.$	MPa
$K_3 = 0.98 (15.7)$	MPa/cm	$G_3 = 1177.$	MPa
$K_4 = 0.98 (4.9)$	MPa/cm	$\tau_{y1} = 0.0005$	MPa
$\tau_{y1} = 4.9 (5.9)$	MPa	$\tau_{y2} = 0.003$	MPa
$\tau_{y2} = 7.4 (9.1)$	MPa		
$\tau_{y3} = 0.5 (0.5)$	MPa		

( ): for Joint Panel

RESULTS OF ANALYSIS AND DISCUSSIONS

Relationship between horizontal force and horizontal displacement

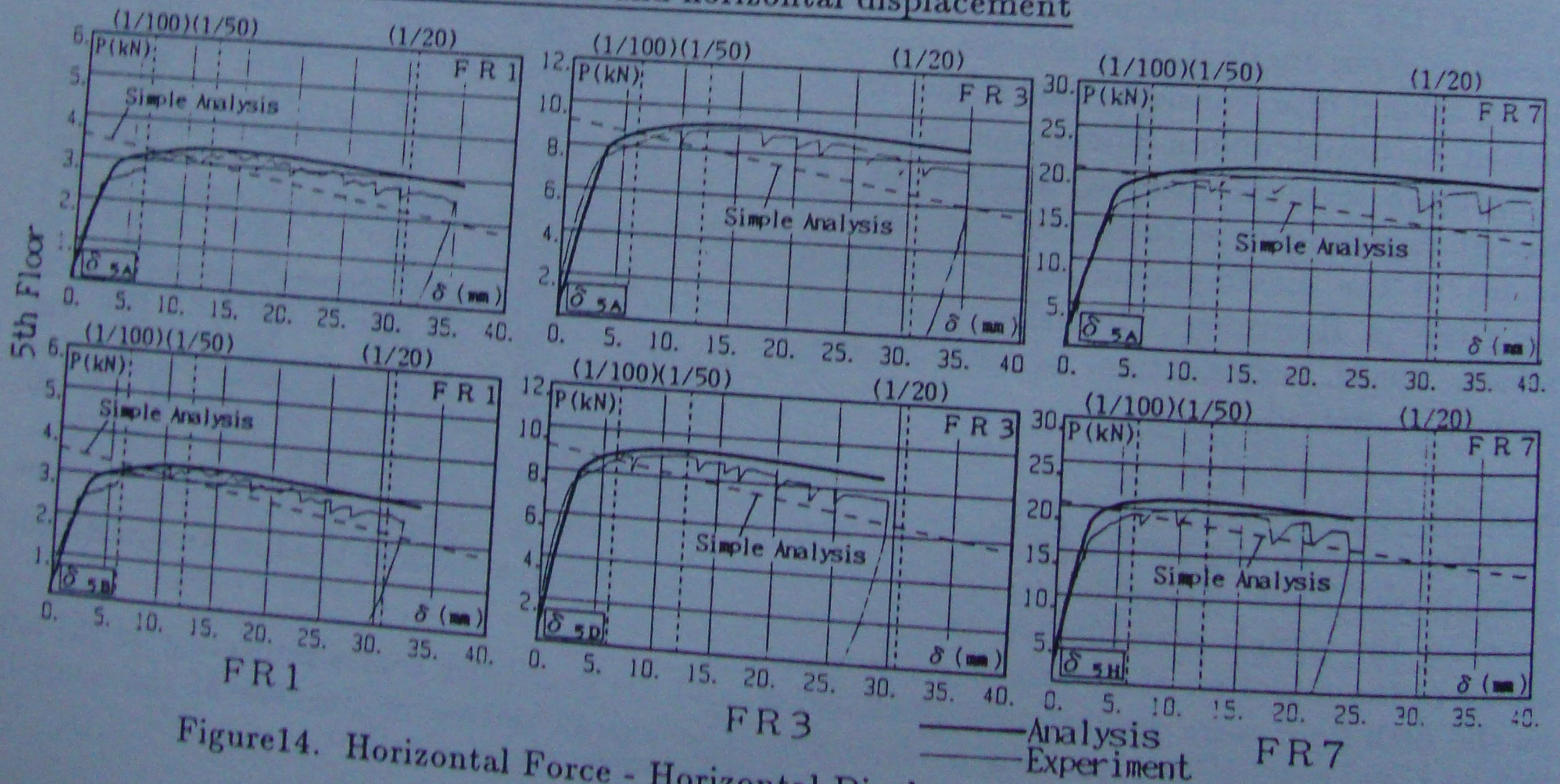


Figure 14. Horizontal Force - Horizontal Displacement Relationships



Fig.14 shows the relationship between horizontal force and horizontal displacement. The bold solid line represents the analysis value and the narrow solid line represents the experimental value. In all specimens, the analysis value is slightly greater than the experimental value, indicating a good correspondence between the two. As in the case of the experimental value, the analysis value indicates a greater strength than simple analysis, in all specimens.

#### Axial elongation of beam

Fig.15 shows the axial elongation of the beams on the second floor when the whole average angle of rotation is 1/20. The experimental values and the analysis values concur well.

#### Relationship between bending moment at beam end and axial force

Fig.16 traces the bending moments generated in the ends of the A-B beams on the second floor and also the axial forces generated in the same beams. The solid line indicates the left end and the broken line indicates the right end. When a compressive force is developed in a beam, the axial force is shown in the positive, and the tensile side of the beam bottom is the positive bending moment. Due to the development of an axial force, the bending moment in the beam end varied with the compressive axial force which also changed along the interaction curve. This bending moment was greater than one with a zero axial force, and from this, it is feasible to explain the fact that increasing resistance strength was seen as in Fig.4.

#### Shearing force of first floor columns

Fig.17 shows the shearing force of the first floor columns with moment diagram for the FR3 when the whole average angle of rotation is 1/200. The shearing force shared by the first floor columns is not uniform because of the effect of axial elongation of the beam on the second floor, particularly a large difference of shear values at both ends.

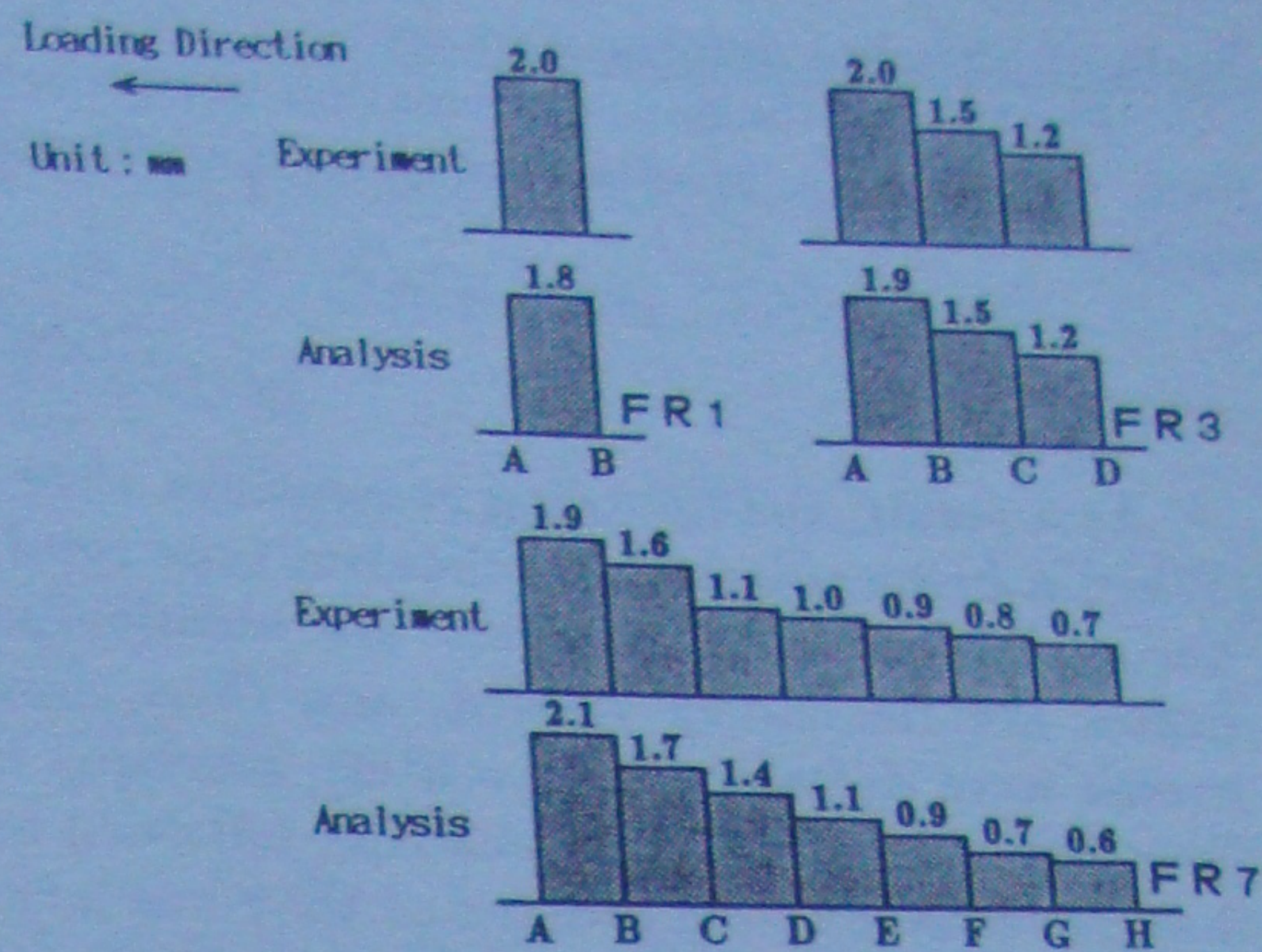


Figure15. Axial Elongation of 2nd Floor Beam at Whole Average Angle of Rotation 1/20

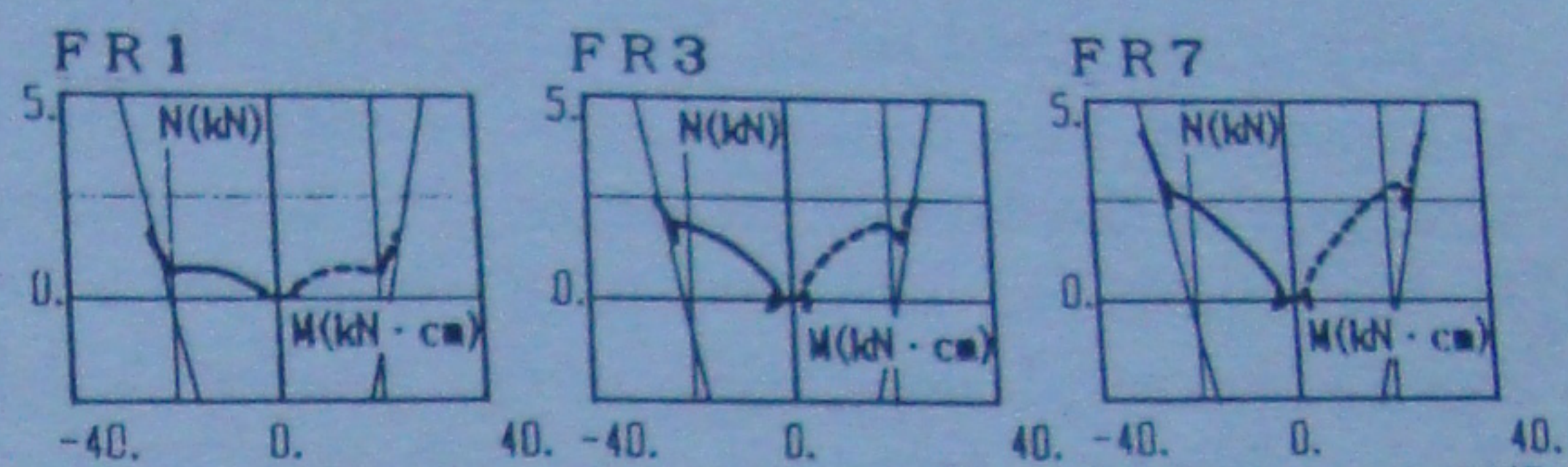


Figure16. Bending Moment at Beam End - Axial Force Relationships on 2nd Floor A-B Beam

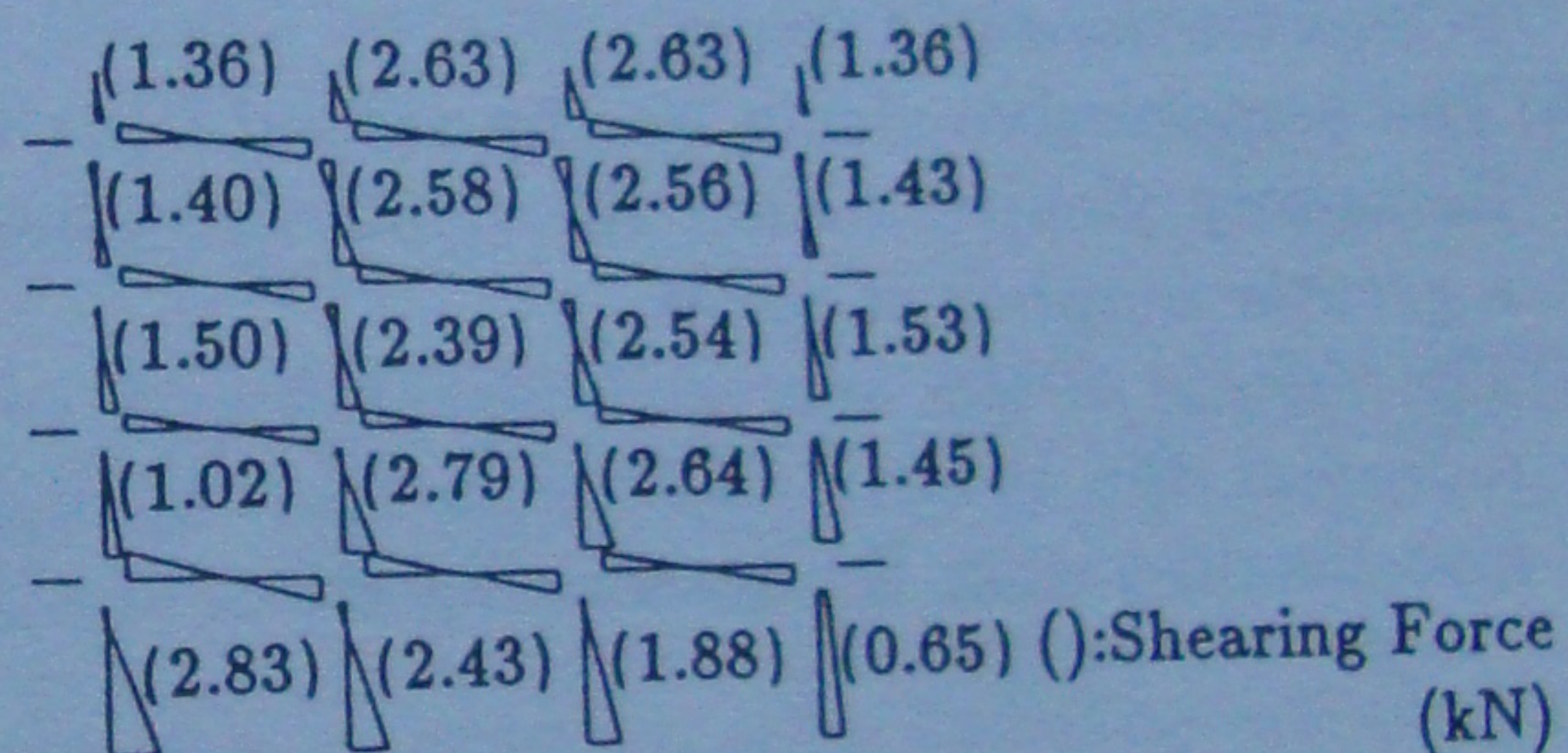


Figure17. Bending Moment Diagram and Shearing Force (FR3)

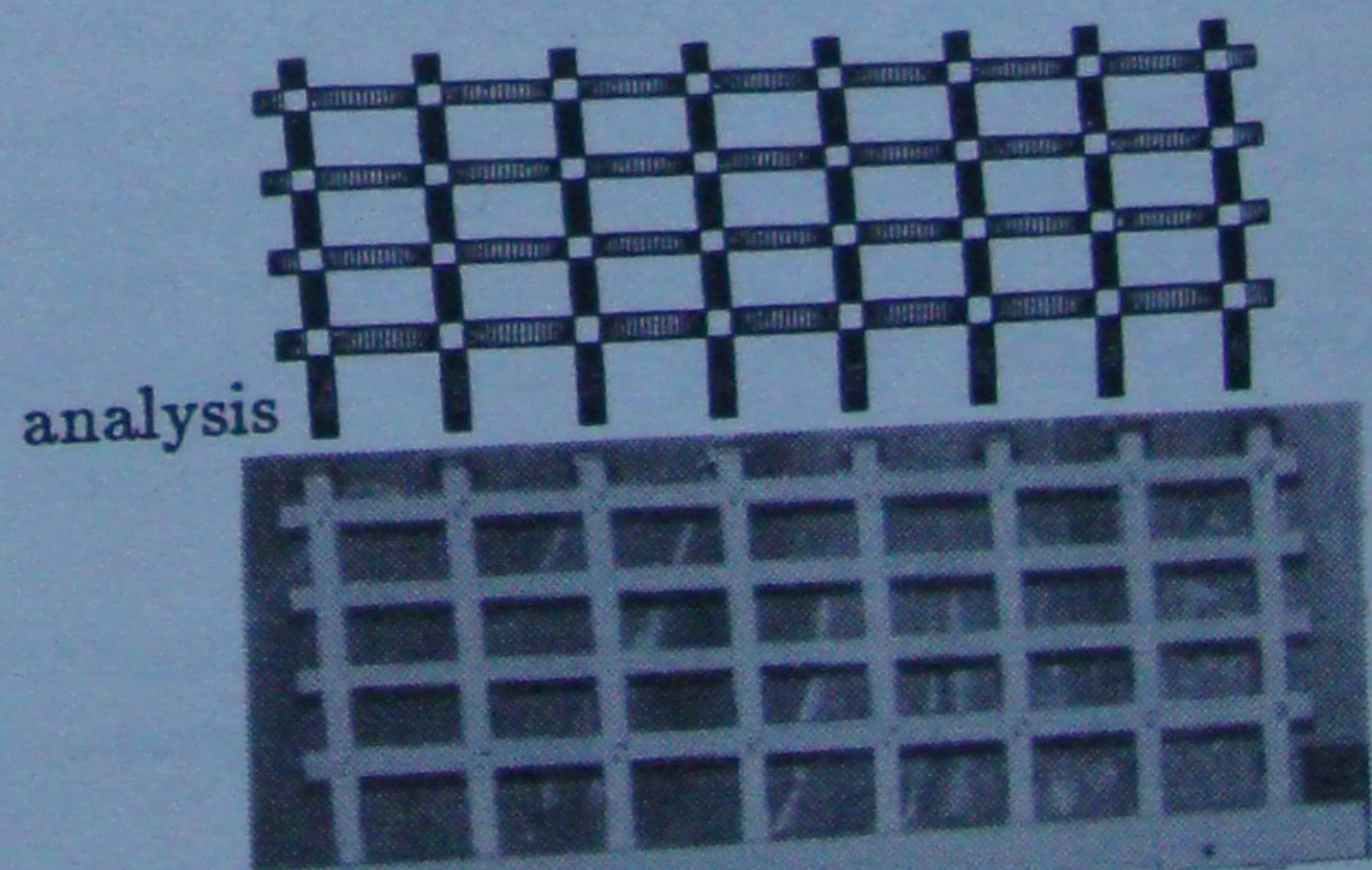


Figure18. State of Ultimate Deformation (FR7)



### State of ultimate deformation

Fig.18 depicts how deformation occurs when the whole average angle of rotation is  $1/20$ . Both the analysis results and the experimental photograph indicate a very close deformation mode.

## CONCLUSIONS

As a result, the following conclusions were achieved.

- 1) When a beam sidesway mechanism type reinforced concrete plane frame is deformed due to a horizontal force, all columns are identically deformed at an early stage. Thereafter, the beams are flexuously yielded, and then elongated in the direction of axis, but the footing beams were deformed only a little, resulting in the varying angles of the columns on the first floor. When the frame is receiving leftward monotonic loading, the angles of the columns on the first floor become greater as they are located closer to the left, resulting in a large difference between those at the left end and the right end. This phenomenon is more noticeable with the increasing number of spans since all the yielding beams tend to elongate.
- 2) All specimens demonstrated that the horizontal strength is greater than the horizontal strength obtained using the ultimate bending moment on the assumption that no axial force is generated in a beam. As the results of the analysis indicated, this was because the bending strength of the beams increased due to a compressive axial force.
- 3) In the reinforced concrete plane frame, the columns are subject to extra forced deformation due to the axial elongation of the beams that have been flexuously yielded, hence the sharing of the shearing forces in the columns at both ends of the frame is greatly varied. This phenomenon is noticeable in the columns on the first and second floors.
- 4) When the reinforced concrete plane frame is monotonically loaded from the right to the left, the elongation of the beams on the left spans on the second floor is greater than that of the beams on the right spans.
- 5) The compressive strain on the second floor beams which tend to axially elongate after being flexuously yielded within the reinforced concrete plane frame is greater in the specimen with a greater number of spans.
- 6) The results of the analysis presented here cover most details of the experimental results, and represent the behavior of the multi-story multi-span reinforced concrete plane frame, taking into consideration the restraining effect of axial deformation of the flexuously yielded beams.

## REFERENCES

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